Surds Advance:-

def:- A surd is an irrational root that cannot be expressed as a rational or whole number.

e.g $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{5}$

$\sqrt{2} \to 1.414\ldots$, $\sqrt{3} \to 1.732\ldots$

$\sqrt{4} = \sqrt{2^{\cancel{2}}} = 2$ ✓

$\sqrt{9} = 3$ ✓

$\sqrt{16} = 4$ ✓

Types of Surds:

4 Types.

1. Pure Surd:- e.g $\sqrt{3}, \sqrt{2}, \sqrt{7}, \sqrt{8}$
2. Mixed Surd:- e.g $2\sqrt{2}, \frac{1}{2}\sqrt{3}, 2\sqrt{7}$
3. Like Surd:- $2\sqrt{3}$, and $5\sqrt{3}$
4. Unlike surds- $2\sqrt{3}$ and $5\sqrt{5}$

Q1: Show that $\sqrt{45} + \sqrt{20} = 5\sqrt{5}$

sol:- L.H.S

$\sqrt{45} + \sqrt{20}$

$= \sqrt{3 \times 3 \times 5} + \sqrt{2 \times 2 \times 5}$

$= \sqrt{3^2 \times 5} + \sqrt{2^2 \times 5}$

$= \sqrt{3^{\cancel{2}}} \times \sqrt{5} + \sqrt{2^{\cancel{2}}} \times \sqrt{5}$

$= 3 \times \sqrt{5} + 2 \times \sqrt{5}$

$= 3\sqrt{5} + 2\sqrt{5}$

$= \sqrt{5}(3+2) \Rightarrow 5\sqrt{5}$

$$\begin{array}{c|c} 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\because \sqrt{\ } = \frac{1}{2}$

$\because \sqrt{3^2} = (3^{\cancel{2}})^{1/\cancel{2}} = 3$

$\because \sqrt{2^2} = (2^{\cancel{2}})^{1/2} = 2$

Q2: express $\frac{2}{\sqrt{3}-1}$ in the form of $\frac{p}{q}$ where p and q are integers.

sol: $\frac{2}{\sqrt{3}-1} \Rightarrow \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\because (a-b)(a+b) = a^2 - b^2$

$a = \sqrt{3}, b = 1$

$= \frac{2(\sqrt{3}+1)}{(\sqrt{3})^{\cancel{2}} - (1)^2}$

$= \frac{2\sqrt{3}+2}{3-1} = \frac{2\sqrt{3}+2}{2} = \frac{\cancel{2}(\sqrt{3}+1)}{\cancel{2}}$

$= \boxed{\sqrt{3}+1}$

$\hookrightarrow p, q$

Q.3: Show that $\frac{\sqrt{20}+\sqrt{80}}{\sqrt{3}}$ can be expressed in the form $\sqrt{a}$ where a is integer.

sol: $\frac{\sqrt{20}+\sqrt{80}}{\sqrt{3}} = \frac{\sqrt{2 \times 2 \times 5} + \sqrt{2 \times 2 \times 2 \times 2 \times 5}}{\sqrt{3}}$

$$\begin{array}{c|c} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$= \frac{\sqrt{2^{\cancel{2}}} \times \sqrt{5} + \sqrt{2^{\cancel{2}}} \times \sqrt{2^{\cancel{2}}} \times \sqrt{5}}{\sqrt{3}}$

$= \frac{2\sqrt{5} + 4\sqrt{5}}{\sqrt{3}}$

$= \frac{2\sqrt{5}+4\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{\sqrt{3}(2\sqrt{5}+4\sqrt{5})}{(\sqrt{3})^{\cancel{2}}}$

$= \frac{2\sqrt{3\times5} + 4\sqrt{3\times5}}{3} = \frac{2\sqrt{15}+4\sqrt{15}}{3}$

$= \frac{\sqrt{15}(2+4)}{3} = \frac{\overset{2}{\cancel{6}}\sqrt{15}}{\cancel{3}_1} = \boxed{2\sqrt{15}}$ Ans:

Q.4: Show that $\frac{4+\sqrt{8}}{\sqrt{2}-1}$ can br written in the form of $a + b\sqrt{2}$, where a, b are integers.

Sol: $\frac{4+\sqrt{8}}{\sqrt{2}-1} \Rightarrow \frac{4+\sqrt{8}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{(4+\sqrt{8})(\sqrt{2}+1)}{(\sqrt{2})^{\cancel{2}} - (1)^2} = \frac{4(\sqrt{2}+1) + \sqrt{8}(\sqrt{2}+1)}{2-1}$

$= \frac{4\sqrt{2} + 4 + \sqrt{16} + \sqrt{8}}{1} = \frac{4\sqrt{2} + 4 + \sqrt{4^{\cancel{2}}} + \sqrt{8}}{1}$

$= \frac{4\sqrt{2} + 4 + 4 + \sqrt{8}}{1} = 4\sqrt{2} + 8 + \sqrt{8}$

$$\begin{array}{c|c} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$8 = 2^2 \times 2$

$= 4\sqrt{2} + 8 + \sqrt{2^2 \times 2}$

$= 4\sqrt{2} + 8 + \sqrt{2^{\cancel{2}}} \times \sqrt{2}$

$a + b\sqrt{2}$

$= 4\sqrt{2} + 8 + 2\sqrt{2}$

$= 8 + 4\sqrt{2} + 2\sqrt{2} \Rightarrow 8 + \sqrt{2}(4+2)$

$= 8 + 6\sqrt{2} \Rightarrow \boxed{8+6\sqrt{2}}$ Ans:

$a = 8, b = 6$