

Recurring Decimals

- e.g. $0.\bar{3}$ OR $0.\dot{3}$
 $0.333333\ldots$
 $0.\overline{27}$ OR $0.\dot{27}$
 $0.27272727\ldots$
- ① pure Recurring decimals
 $0.\bar{3}$, $0.\overline{27}$, $0.\overline{25}$
 $0.\overline{45}$, $0.\overline{37}$
- ② Mix Recurring decimals.
 $0.1\bar{3} \Rightarrow 0.13333\ldots$
 $0.1\overline{73} \Rightarrow 0.173\overline{73}\ldots$

① Terminating decimals

0.25, 1.75, 1.35, 0.45

② Recurring decimals

$\hookrightarrow 0.\overline{27} = 0.272727\ldots$

$0.\dot{3} = 0.333\ldots$

Recurring decimals numbers:

① $0.\dot{3} = 0.3333\ldots$

② $0.\bar{3} = 0.3333\ldots$

e.g. $0.\dot{24} = 0.2424\ldots$

$0.\overline{24} = 0.2424\ldots$

$0.\dot{125} = 0.125125125\ldots$

$0.\overline{125} = "$

Example 1: Convert $0.\bar{3}$ to a fraction

sol Let $x = 0.\bar{3} \rightarrow$ ①

multiply both sides by 10.

$$10x = 10 \times 0.\bar{3} \rightarrow 10 \times 0.3333\ldots$$

$$10x = 3.\bar{3} \text{ OR } 3.3333\ldots \rightarrow$$
 ②

Subtract eq ① from eq ②

$$10x - x = 3.\bar{3} - 0.\bar{3}$$

$$9x = 3 \Rightarrow$$

$$x = \frac{3}{9}$$

$0.\bar{3}$

$\leftarrow$

Example 2: Convert $0.0\dot{7}$ to a fraction

sol Let $x = 0.0\dot{7} \rightarrow$ ①

multiply both sides by 10.

$$10x = 10 \times 0.0\dot{7}$$

$$10x = 0.7\bar{7} \rightarrow$$
 ②

Multiply eq ② with 10.

$$10 \times 10x = 10 \times 0.7\bar{7}$$

$$100x = 7.\bar{7} \rightarrow$$
 ③

subtract eq ② from eq ③

$$100x - 10x = 7.\bar{7} - 0.7\bar{7}$$

$$90x = 7 \Rightarrow$$

$$x = \frac{7}{90}$$

$0.0\dot{7}$

$\checkmark$

Example 3: Convert $0.\overline{27}$ to a fraction

sol Let $x = 0.\overline{27} \rightarrow$ ①

multiply both sides by 100

$$100x = 100 \times 0.\overline{27}$$

$$100x = 27.\overline{27} \rightarrow$$
 ②

subtract eq ① from eq ②

$$100x - x = 27.\overline{27} - 0.\overline{27}$$

$$99x = 27 \Rightarrow x = \frac{27}{99} = \frac{9}{33} = \frac{3}{11}$$

$$x = \frac{3}{11}$$